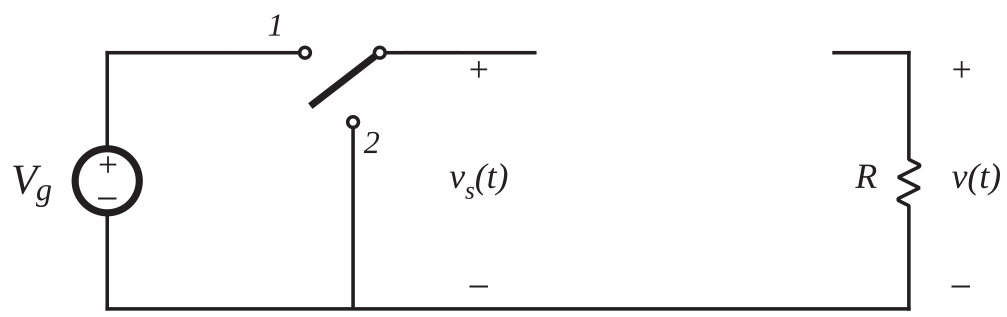
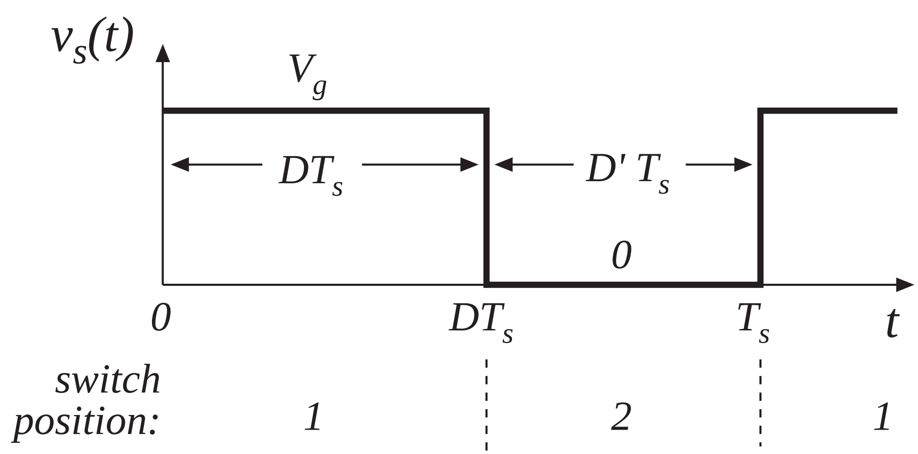
2.1 Introduction
Buck converter

SPDT switch changes dc component



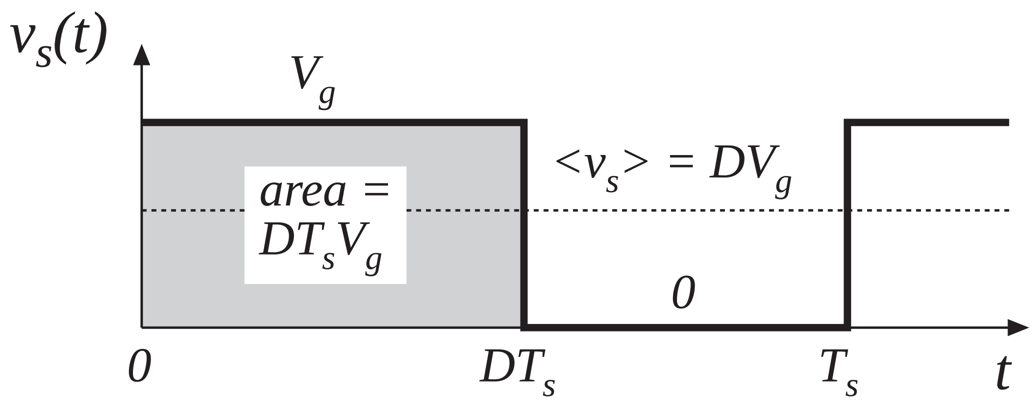
Switch output voltage waveform



Duty cycle *D:*
$0 \leq D \leq 1$

complement D':
D' = 1 - D

Dc component of switch output voltage

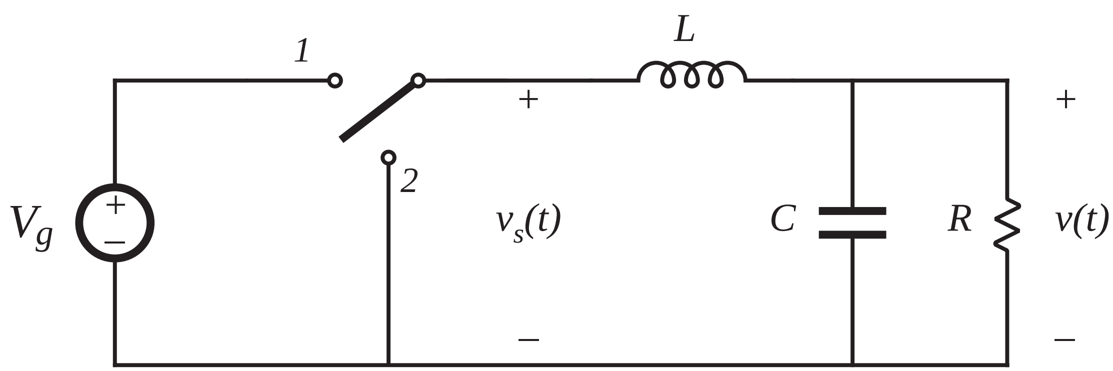


Fourier analysis: Dc component = average value

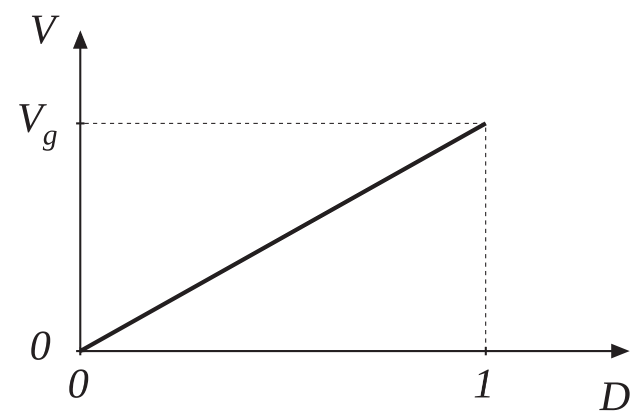
$$\left\langle v_s \right\rangle = \frac{1}{T_s}\int_0^{T_s} v_s(t)\,dt$$

$$\left\langle v_s \right\rangle = \frac{1}{T_s}\left(DT_sV_g\right) = DV_g$$

Insertion of low-pass filter to remove switching harmonics and pass only dc component



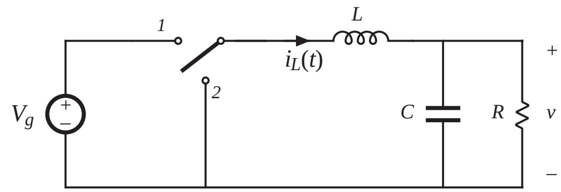
$$v \approx \langle v_s \rangle = DV_g$$

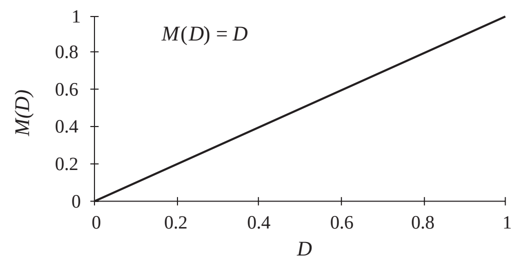


Three basic dc-dc converters

Buck

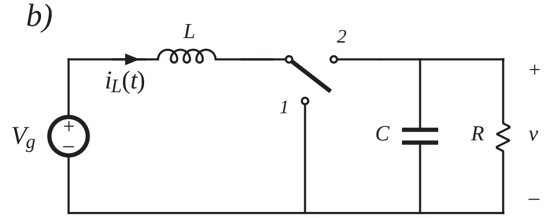
a)

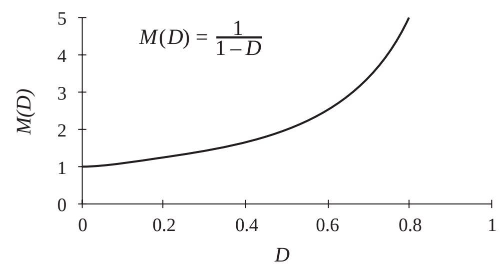




Boost

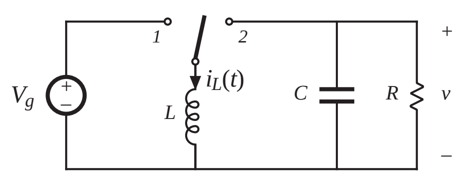
b)

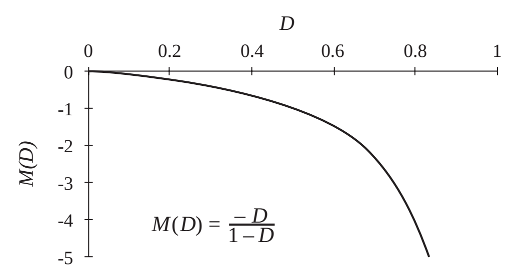




Buck-boost

c)





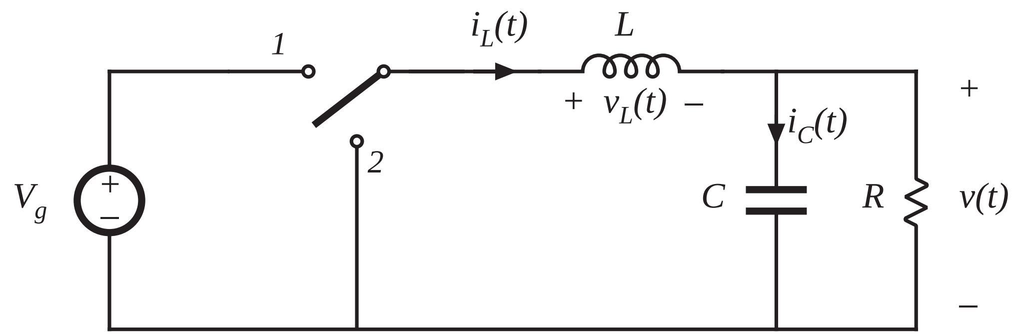
Objectives of this chapter

- Develop techniques for easily determining output voltage of an arbitrary converter circuit
- Derive the principles of *inductor volt-second balance* and *capacitor charge (amp-second) balance*
- Introduce the key *small ripple approximation*
- Develop simple methods for selecting filter element values
- Illustrate via examples

2.2. Inductor volt-second balance, capacitor charge balance, and the small ripple approximation

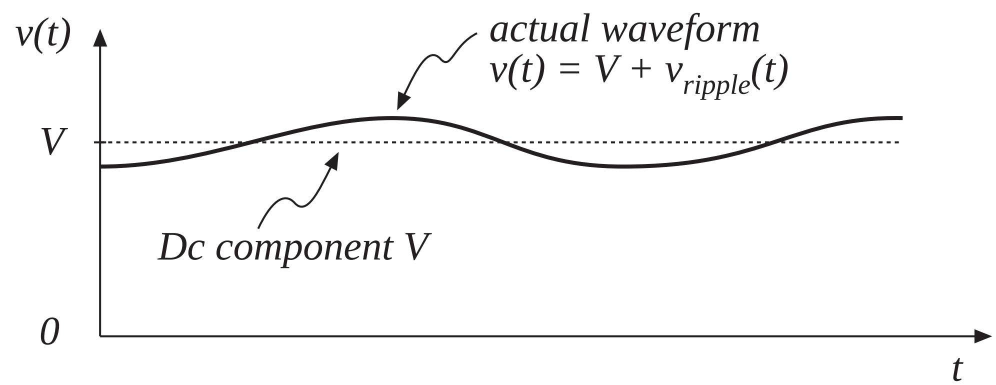
Actual output voltage waveform, buck converter

Buck converter containing practical low-pass filter



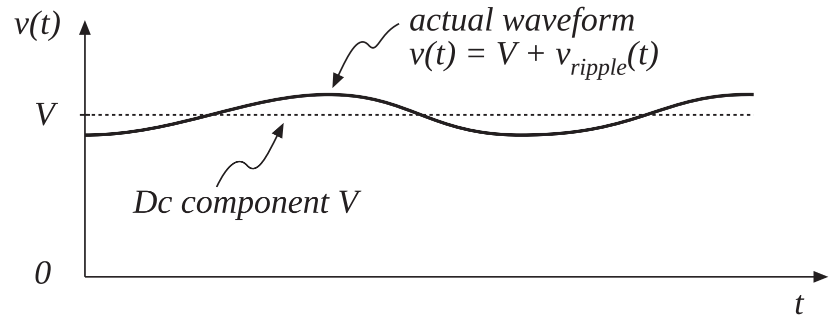
Actual output voltage waveform

$$v(t) = V + v_{ripple}(t)$$



The small ripple approximation

$$v(t) = V + v_{ripple}(t)$$



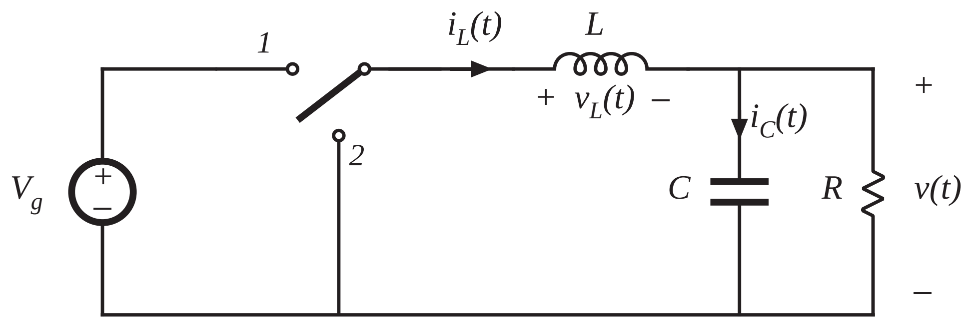
In a well-designed converter, the output voltage ripple is small. Hence, the waveforms can be easily determined by ignoring the ripple:

$$\left\| v_{ripple} \right\| << V$$

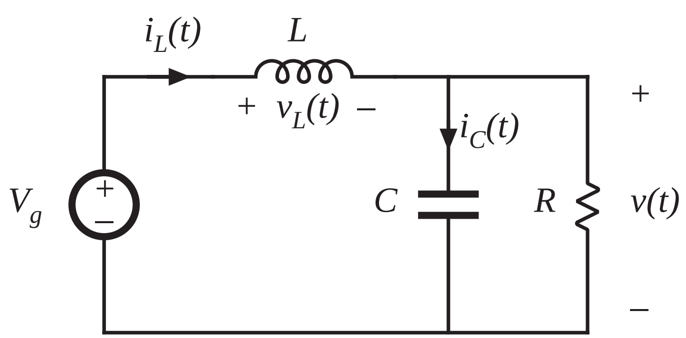
$$v(t) \approx V$$

Buck converter analysis: inductor current waveform

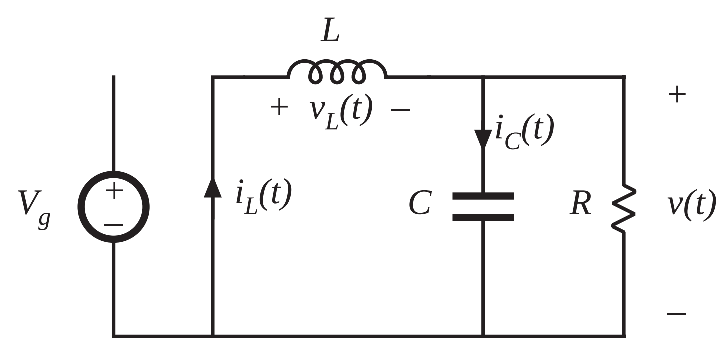
original converter



switch in position 1



switch in position 2



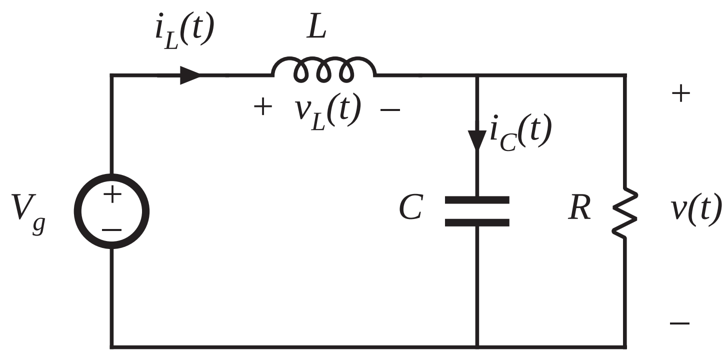
Inductor voltage and current
Subinterval 1: switch in position 1

Inductor voltage

$$v_L = V_g - v(t)$$

Small ripple approximation:

$$v_L \approx V_g - V$$



Knowing the inductor voltage, we can now find the inductor current via

$$v_L(t) = L \frac{di_L(t)}{dt}$$

Solve for the slope:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} \approx \frac{V_g - V}{L}$$

$\Rightarrow$ *The inductor current changes with an essentially constant slope*

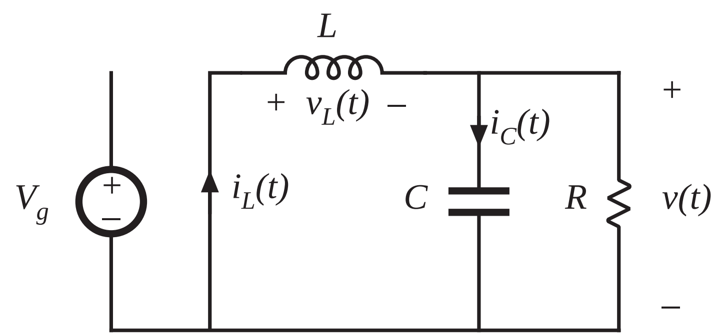
Inductor voltage and current
Subinterval 2: switch in position 2

Inductor voltage

$$v_L(t) = -v(t)$$

Small ripple approximation:

$$v_L(t) \approx -V$$



Knowing the inductor voltage, we can again find the inductor current via

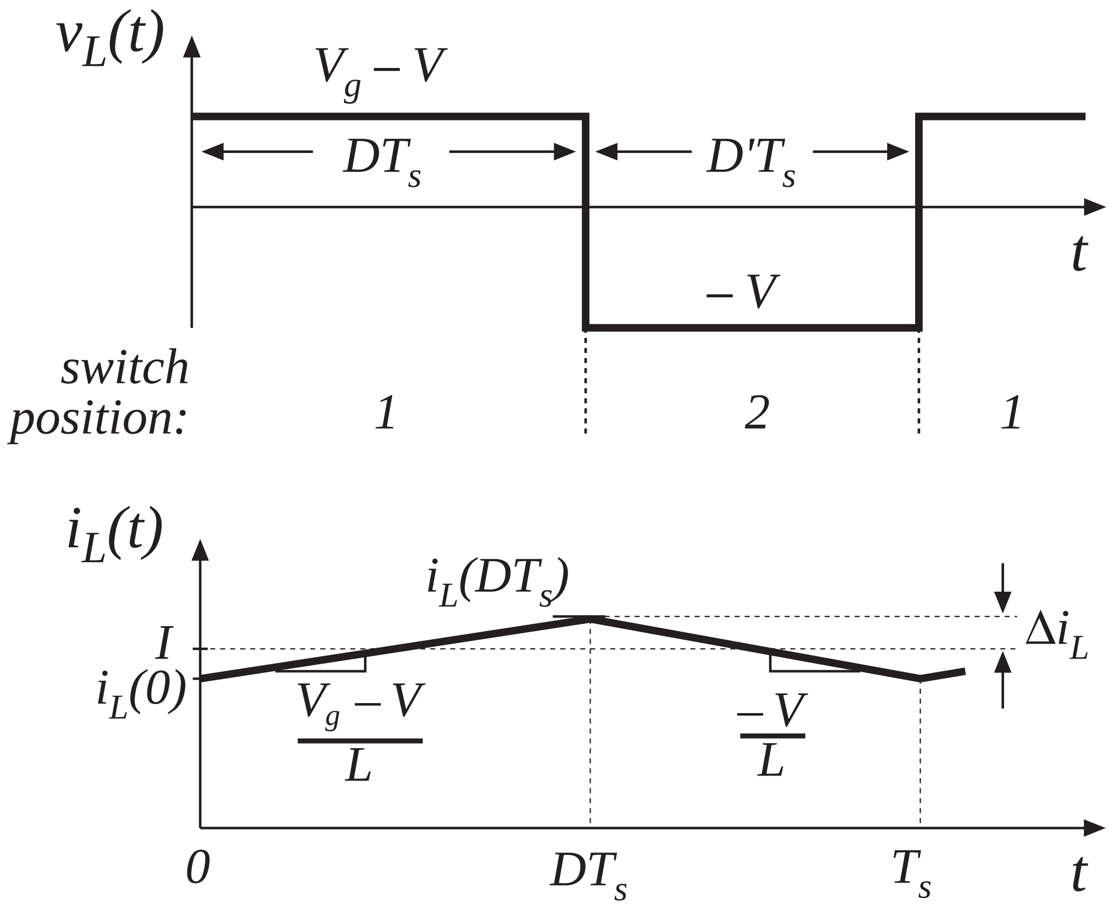
$$v_L(t) = L\,\frac{di_L(t)}{dt}$$

Solve for the slope:

$$\frac{di_L(t)}{dt} \approx -\frac{V}{L}$$

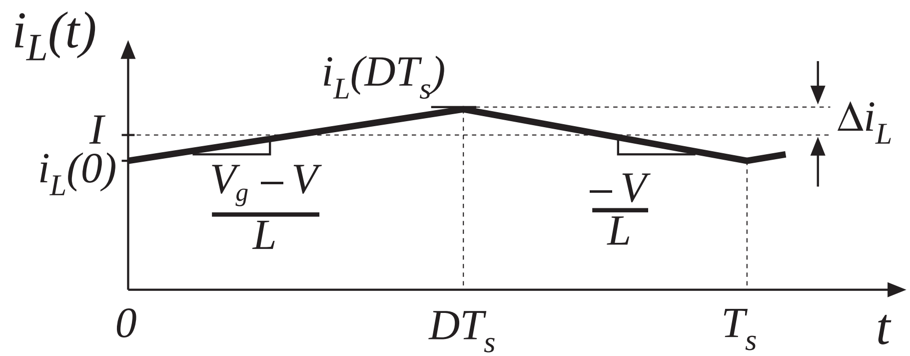
⇒ *The inductor current changes with an essentially constant slope*

Inductor voltage and current waveforms



$$v_L(t) = L\,\frac{di_L(t)}{dt}$$

Determination of inductor current ripple magnitude

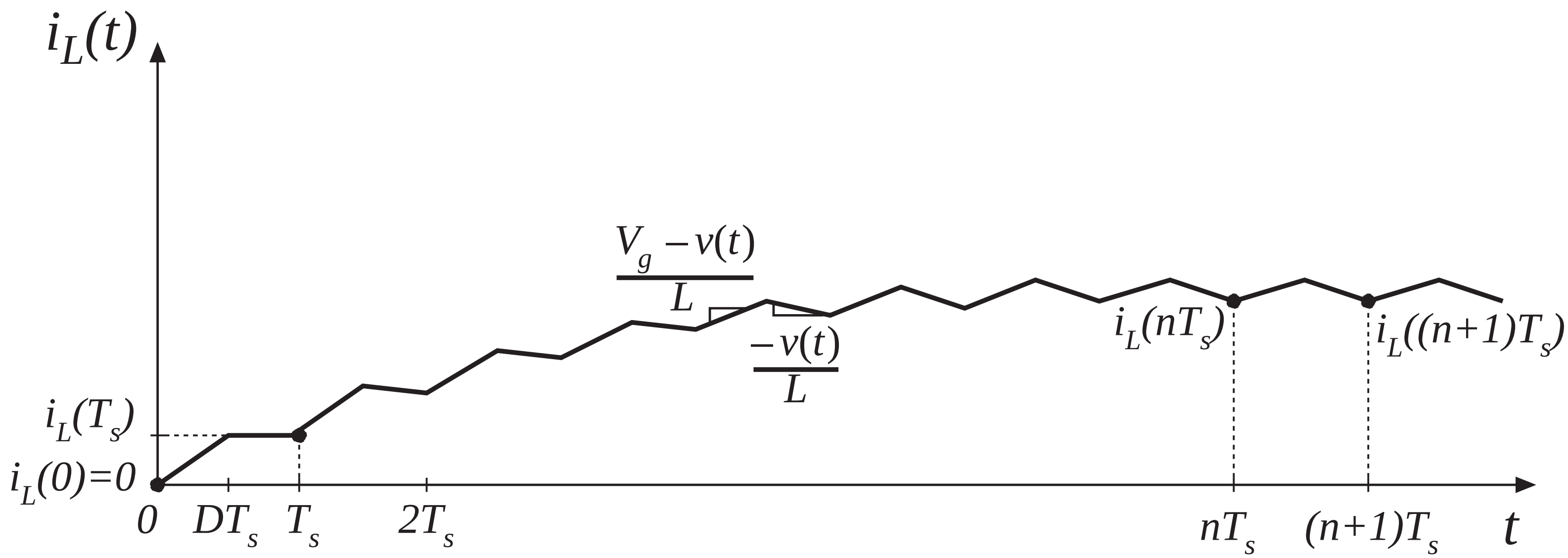


$$(\textit{change in } i_L) = (\textit{slope})(\textit{length of subinterval})$$

$$\left(2\Delta i_L\right) = \left(\frac{V_g - V}{L}\right)\left(DT_s\right)$$

$$\Rightarrow \qquad \Delta i_L = \frac{V_g - V}{2L} DT_s \qquad\qquad L = \frac{V_g - V}{2\Delta i_L} DT_s$$

Inductor current waveform during turn-on transient



When the converter operates in equilibrium:

$$i_L((n+1)T_s) = i_L(nT_s)$$

The principle of inductor volt-second balance: Derivation

Inductor defining relation:

$$v_L(t) = L\,\frac{di_L(t)}{dt}$$

Integrate over one complete switching period:

$$i_L(T_s) - i_L(0) = \frac{1}{L}\int_0^{T_s} v_L(t)\,dt$$

In periodic steady state, the net change in inductor current is zero:

$$0 = \int_0^{T_s} v_L(t)\,dt$$

Hence, the total area (or volt-seconds) under the inductor voltage waveform is zero whenever the converter operates in steady state.

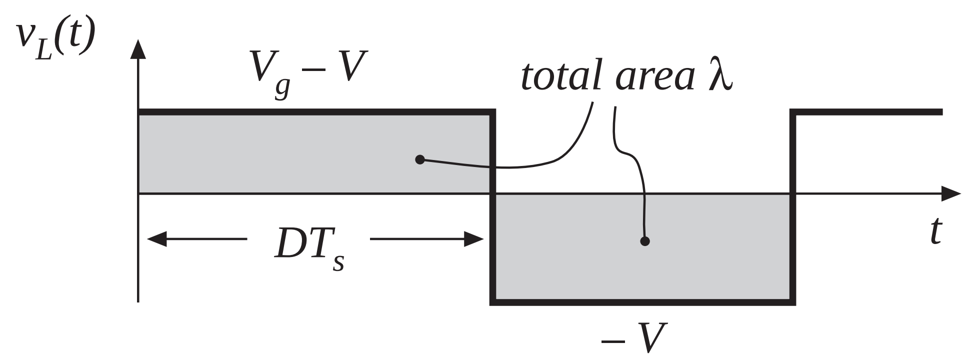
An equivalent form:

$$0 = \frac{1}{T_s}\int_0^{T_s} v_L(t)\,dt = \left\langle v_L \right\rangle$$

The average inductor voltage is zero in steady state.

Inductor volt-second balance: Buck converter example

Inductor voltage waveform, previously derived:



Integral of voltage waveform is area of rectangles:

$$\lambda = \int_0^{T_s} v_L(t)\,dt = (V_g - V)(DT_s) + (-V)(D'T_s)$$

Average voltage is

$$\langle v_L \rangle = \frac{\lambda}{T_s} = D(V_g - V) + D'(-V)$$

Equate to zero and solve for *V:*

$$0 = DV_g - (D + D')V = DV_g - V \qquad \Rightarrow \qquad V = DV_g$$

The principle of capacitor charge balance: Derivation

Capacitor defining relation:

$$i_C(t) = C\,\frac{dv_C(t)}{dt}$$

Integrate over one complete switching period:

$$v_C(T_s) - v_C(0) = \frac{1}{C}\int_0^{T_s} i_C(t)\,dt$$

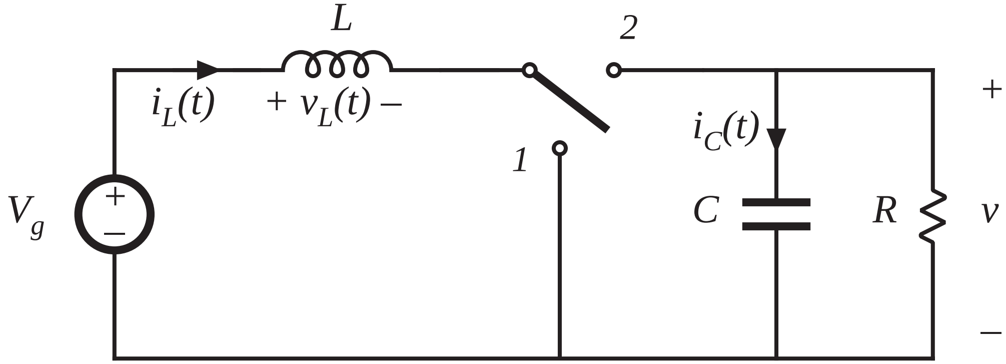
In periodic steady state, the net change in capacitor voltage is zero:

$$0 = \frac{1}{T_s}\int_0^{T_s} i_C(t)\,dt = \left\langle i_C \right\rangle$$

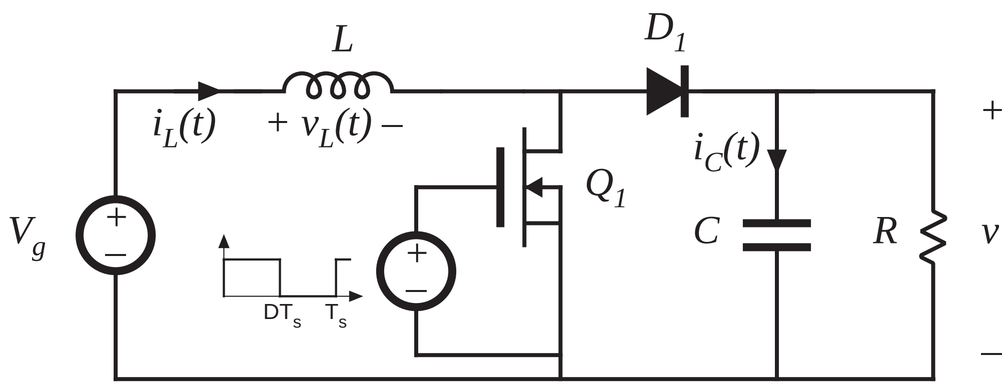
Hence, the total area (or charge) under the capacitor current waveform is zero whenever the converter operates in steady state. The average capacitor current is then zero.

2.3 Boost converter example

Boost converter with ideal switch

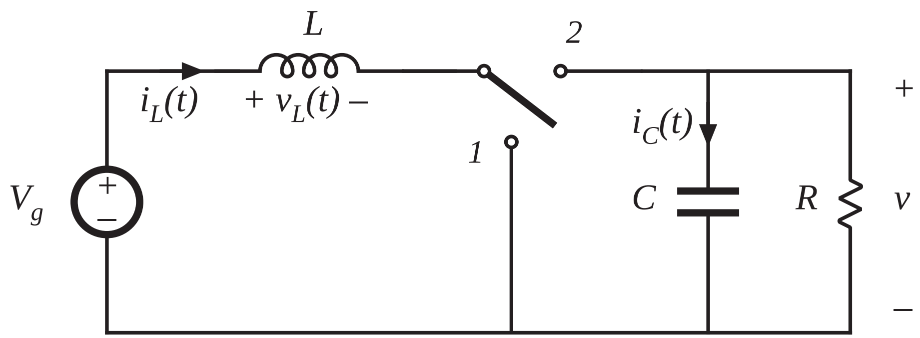


Realization using power MOSFET and diode

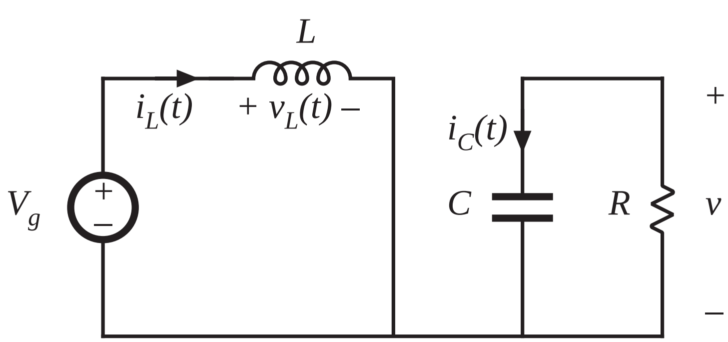


Boost converter analysis

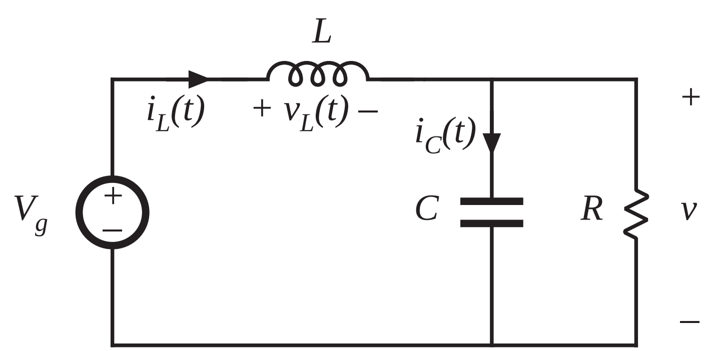
original converter



switch in position 1



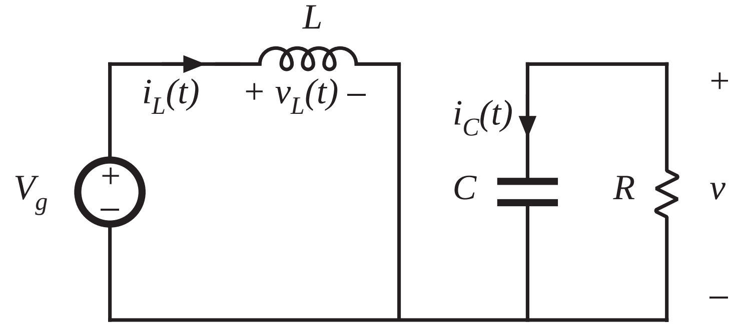
switch in position 2



Subinterval 1: switch in position 1

Inductor voltage and capacitor current

$$v_L = V_g$$
$$i_C = -\, v \,/\, R$$



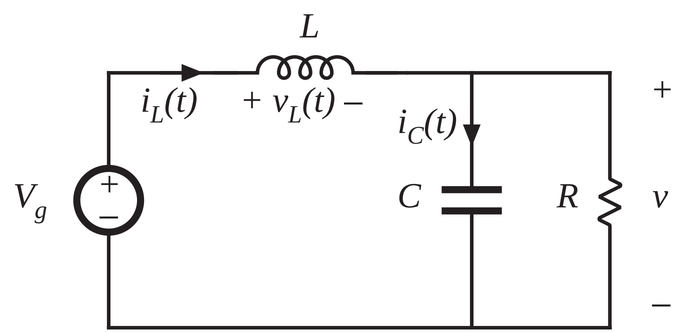
Small ripple approximation:

$$v_L = V_g$$
$$i_C = -\, V \,/\, R$$

Subinterval 2: switch in position 2

Inductor voltage and capacitor current

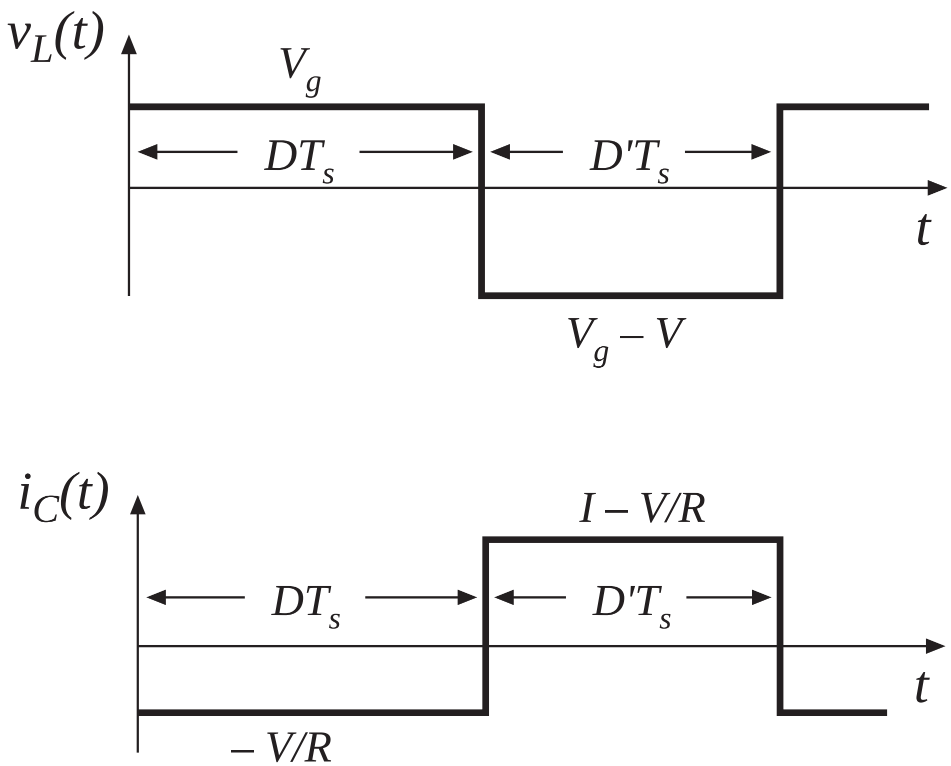
$$v_L = V_g - v$$
$$i_C = i_L - v / R$$



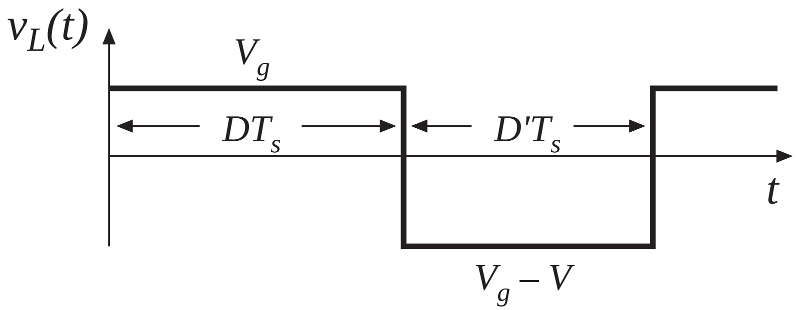
Small ripple approximation:

$$v_L = V_g - V$$
$$i_C = I - V / R$$

Inductor voltage and capacitor current waveforms



Inductor volt-second balance



Net volt-seconds applied to inductor over one switching period:

$$\int_0^{T_s} v_L(t)\, dt = (V_g)\, DT_s + (V_g - V)\, D'T_s$$

Equate to zero and collect terms:

$$V_g\,(D + D') - V\,D' = 0$$

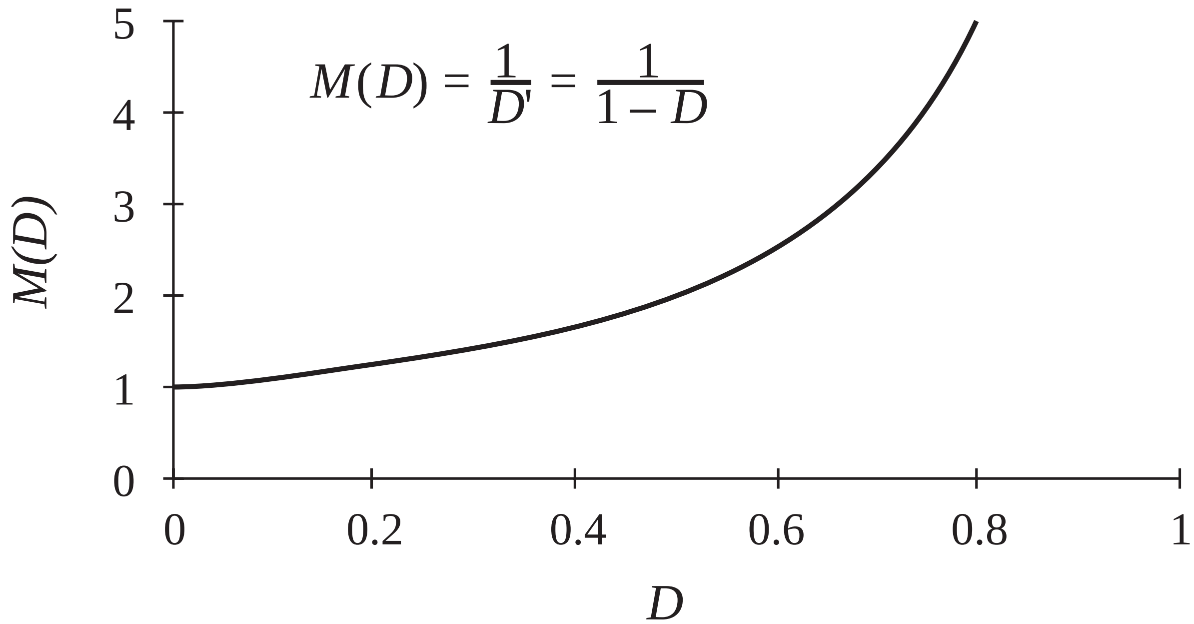
Solve for V:

$$V = \frac{V_g}{D'}$$

The voltage conversion ratio is therefore

$$M(D) = \frac{V}{V_g} = \frac{1}{D'} = \frac{1}{1 - D}$$

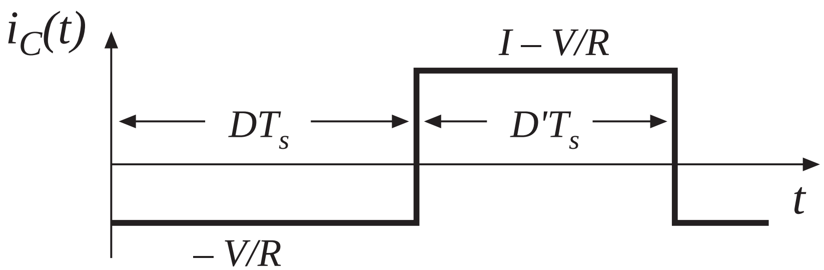
Conversion ratio *M(D)* of the boost converter



$$M(D) = \frac{1}{D'} = \frac{1}{1-D}$$

Determination of inductor current dc component

Capacitor charge balance:



$$\int_0^{T_s} i_C(t)\,dt = (-\frac{V}{R})\,DT_s + (I - \frac{V}{R})\,D'T_s$$

Collect terms and equate to zero:

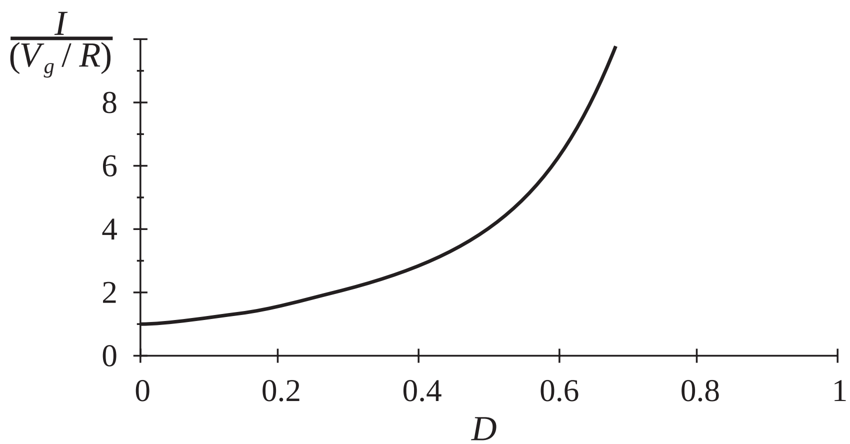
$$-\frac{V}{R}(D + D') + I\,D' = 0$$

Solve for *I:*

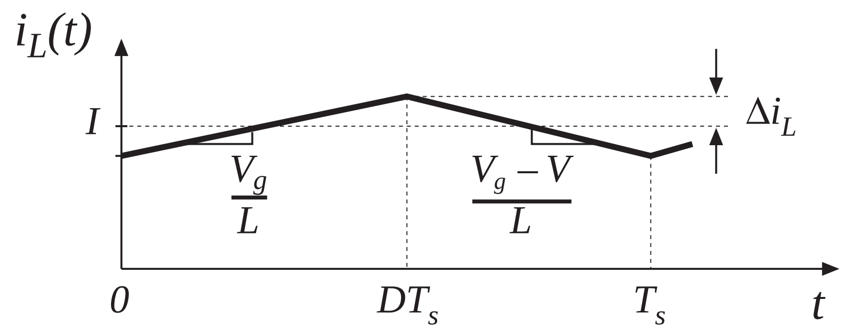
$$I = \frac{V}{D'\,R}$$

Eliminate *V* to express in terms of V_g:

$$I = \frac{V_g}{D'^2\,R}$$



Determination of inductor current ripple



Inductor current slope during subinterval 1:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g}{L}$$

Inductor current slope during subinterval 2:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g - V}{L}$$

Change in inductor current during subinterval 1 is *(slope) (length of subinterval):*

$$2\Delta i_L = \frac{V_g}{L} DT_s$$

Solve for peak ripple:

$$\Delta i_L = \frac{V_g}{2L} DT_s$$

- Choose *L* such that desired ripple magnitude is obtained

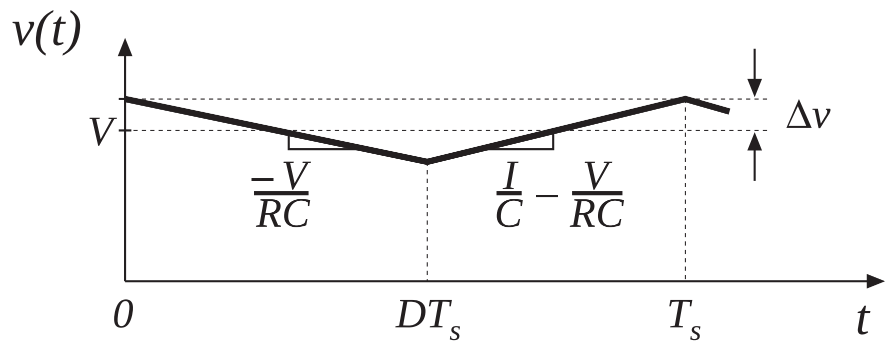
Determination of capacitor voltage ripple

Capacitor voltage slope during subinterval 1:

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{-V}{RC}$$

Capacitor voltage slope during subinterval 2:

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{I}{C} - \frac{V}{RC}$$



Change in capacitor voltage during subinterval 1 is *(slope) (length of subinterval):*

$$-2\Delta v = \frac{-V}{RC} DT_s$$

Solve for peak ripple:

$$\Delta v = \frac{V}{2RC} DT_s$$

- Choose *C* such that desired voltage ripple magnitude is obtained
- In practice, capacitor *equivalent series resistance* (esr) leads to increased voltage ripple